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Coincident Dp-branes in codimension two

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Abstract

We present the target-space supersymmetric and the world-volume κ symmetric invariant action for N coincident D0 branes in codimension two i.e. in space-time dimension three. Our analysis is restricted to flat space-time and first order formalism. This is an extension of Sorokin's proposal for the case of co-dimension one. We make brief remarks on the implementation of this proposal for high branes as well as for higher dimensional space-time.

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1 Introduction

In recent years the importance of Dp branes have been realized through the studies of black hole entropy, AdS/CFT correspondence, tachyon condensation etc. Hence, it has been necessary to study the dynamics of these branes in various configurations. The dynamics of a single brane (both BPS and non-BPS) is known to be described by the sum of a Born-Infeld type action and a Chern-Simon action. The various degrees of freedom described by such an action include a $U(1)$ valued gauge field, transverse scalars, induced metric, induced Kalb-Ramond field on the world volume of the Dp brane coming from the NS-NS sector, and antisymmetric field of various ranks coming from the R-R sector. The supersymmetric and kappa symmetric invariant action describing a single BPS D-brane is known for quite some time [1]. The simplest generalization of this is the system of N number of coincident Dp branes, instead of a single brane. It is known that for such a system one gets a non-abelian theory with a gauge group $U(N)$[2] and the fields that live on the brane take values in the adjoint representation of $U(N)$. It is also known that these branes are charged under R-R fields [3] and one expects that this system should also preserve half of the supersymmetries just like a single BPS brane. However, to show that this is indeed true one needs to show that the world volume action is in fact invariant under kappa symmetry as in the case for a single BPS Dp-brane. But such a proof has not been achieved so far. The main obstacle is that unlike a single brane where the kappa symmetry is realized as an abelian symmetry, for the case of multiple coincident branes, this symmetry is enhanced to a non-abelian symmetry.

The dynamics of the N coincident Dp branes is described in great detail in [4], which is derived by starting with a space filling brane but subsequent T-dualisation gives the desired action. The bosonic parts of the world volume action for N coincident Dp-branes are given as:

$$S_{BI} = -T_p \int d^{p+1}\sigma STr\left(e^{-\phi}\sqrt{-det(P[E_{ab} + E_{ai}(Q^{-1} - \delta)^{ij}E_{jb}] + \lambda F_{ab})det(Q^i_j)}\right), \tag{1}$$

with

$$E_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}, \qquad and \qquad Q^i_j = \delta^i_j + i\lambda[\Phi^i, \Phi^k]E_{kj}, \tag{2}$$

and

$$S_{CS} = \mu_p \int STr\left(P[e^{i\lambda i_\Phi i_\phi}(\sum_n C^{(n)}e^B)]e^{\lambda F}\right), \tag{3}$$

where $G_{\mu\nu}$, $B_{\mu\nu}$, ϕ, are the metric, Kalb-Ramond and the dilaton field coming from the NS-NS sector, respectively. F_{ab} is the $U(N)$ valued field strength that live on the world volume of the Dp brane and $C^{(n)}$'s are the rank n antisymmetric field coming from R-R sector. P is the pull-back which acts on the bulk fields and brings it onto the world volume of the brane. Φ^i's are the $U(N)$ valued scalars, and STr stands for the symmetrised average over all orderings of the $U(N)$ valued objects and then taking the trace. The pull back is defined as

$$P[E]_{ab} = E_{ab} + E_{ai} D_b \Phi^i + E_{ib} D_a \Phi^i + D_a \Phi^i D_b \Phi^j E_{ij} \tag{4}$$

where D_a are the covariant derivatives with respect to the world volume coordinates and the gauge fields being the connection. The expression of the pull-back, as written in eq.(4), is expressed in the static gauge choice, namely $X^a = \sigma^a$, and $X^i = \Phi^i$, with the choice $\lambda = 1$, i.e. $p+1$ coordinates of the target space are identified with the world volume coordinates σ^a and the indices i, j denotes the target space coordinates transverse to the Dp branes. Hence the space-time symmetry is now broken to world volume symmetry times the symmetry in the transverse directions to the branes. Since the above action is written in the static gauge, it implies that it cannot be world volume reparametrisation invariant and invariant under the full target space diffeomorphism, except for the case of space filling branes. In order to write an action which is both supersymmetric and κ symmetric invariant, we have to make it both world volume reparametrisation invariant and target space diffeomorphism invariant, otherwise it might be difficult to write down an action which is both supersymmetric and κ symmetric invariant. In a recent paper Sorokin has proposed a first order action for coincident D0 branes in codimension one which is shown to have both the world volume and space-time diffeomorphism invariance [5]. In this letter we extend this proposal to coincident D0 branes for codimension two. We find that even for this minimal extension, the proposal becomes too nontrivial to implement.

Before we review the above construction, it is useful to note an interesting point about the Chern-Simon action, namely the N coincident Dp branes can couple to higher rank R-R fields through $e^{ii_\Phi i_\Phi}$, [4], and which in turn gives rise to various fuzzy surfaces, like S^2 and S^4 [6] and also studied in [7].

In [8] a possible supersymmetric and kappa symmetric invariant action has been constructed but up to order F^2 in the field strength and in [9] the supersymmetric and fermion couplings have been considered in the context of Matrix theory of D0 branes.

The construction of [5] is to consider coincident N Dp branes as a single brane configuration, and call it NDp-brane. The transverse coordinates of this single brane is described by $x^i(\sigma)$, which is given as the trace of the $U(N)$ valued scalars Φ^i, i.e.

$$x^i(\sigma) = \frac{1}{N} Tr \Phi^i(\sigma). \tag{5}$$

Now this transverse coordinate x^i together with the world volume coordinate σ^a represents the coordinates of the single brane in the target space-time, i.e. $x^\mu(\sigma) = (\sigma^a, x^i(\sigma))$ with $\mu = 0, 1, \ldots, D-1$, in a D dimensional target space in the static gauge. In order to make the world volume theory of that single brane diffeomorphism invariant, let's introduce p+1 coordinates, $x^a(\sigma)$, as the world volume coordinates of the (single) NDp-brane, i.e.

$$x^\mu(\sigma) = (x^a(\sigma), x^i(\sigma)). \tag{6}$$

Then the $U(N)$ vector fields $A_a(\sigma)$ and the traceless scalars

$$\phi^i(\sigma) \equiv \Phi^i(\sigma) - x^i(\sigma)I, \qquad \in \quad SU(N), \tag{7}$$

takes values in $SU(N)$, and are being considered as pure world volume vector fields and scalar fields living on the NDp-brane. At this stage, we are assuming that this single NDp-brane action is invariant under only one kappa symmetry as rest N-1 kappa symmetries have been gauge fixed by construction. This feature will be more explicit in the next section.

The action of N coincident Dp-branes, eq.(1), gives us a complicated form after imposing the world volume reparametrisation invariance in a generic background. Hence, to avoid complications, we shall deal with the action in co-dimension one and two in flat spacetime, as the mass shell condition becomes complicated for higher codimension. The meaning of co-dimension one and two is that the dimension of the target space for a single NDp-brane becomes $p+2$, and $p+3$ respectively.

In section 2 we shall write down the world volume reparametrisation invariant action of N Dp branes in co-dimension d in flat spacetime, and in section 3 we shall write down the world volume reparametrisation invariant and kappa symmetric invariant action of N D0 branes in co-dimension 1 by implementing the proposal of [5]. In section 4 we shall do it for co-dimension 2 i.e. in space-time dimension 3. Then we shall conclude in section 5 and in the appendix we have derived the mass shell condition for the co-dimension two case.

2 N Dp branes in codimension d

In codimension d the Born-Infeld action eq. (1) becomes, in the flat background, i.e. $B_{\mu\nu} = 0$, $G_{\mu\nu} = \eta_{\mu\nu}$, $\phi = 0$, and in the static gauge

$$\begin{aligned} S = -T_p \int d^{p+1}\sigma STr \quad & \Big[(-det(\partial_a\sigma^c\partial_b\sigma^d\eta_{cd} + D_a\Phi^i D_b\Phi^j\eta_{ij} \\ & + D_a\Phi^k D_b\Phi^l\eta_{ki}(Q^{-1}-\delta)^{ij}\eta_{lj} + F_{ab}))det(Q^i_j)\Big]^{\frac{1}{2}}, \end{aligned} \tag{8}$$

with

$$Q^i_j = \delta^i_j + i[\Phi^i, \Phi^k]\eta_{kj}. \tag{9}$$

Let's restore the world volume reparametrisation invariance by writing $\sigma^a = x^a(\sigma)$, using eq. (6) and eq. (7) and substituting it in eq.(8). This gives us the following action:

$$S = -T_p \int d^{p+1}\sigma STr \quad \Big[(-det(\partial_a x^\mu \partial_b x^\nu \eta_{\mu\nu} + \partial_a x^\mu D_b \phi^j \eta_{\mu j} + D_a \phi^i \partial_b x^\nu \eta_{i\nu} + D_a \phi^i D_b \phi^j \eta_{ij} + D_a \phi^k D_b \phi^l L_{kl} + \partial_a x^\mu \partial_b x^\nu L_{\mu\nu} + \partial_a x^\mu D_b \phi^l L_{\mu l} + D_a \phi^k \partial_b x^\nu L_{k\nu} + F_{ab}))det(Q^i_j)\Big]^{\frac{1}{2}}, \quad (10)$$

where $L_{AB} = \eta_{Ai}(Q^{-1} - \delta)^{ij}\eta_{jB}$, and A, B can take values (a,i). The above action has a complicated form and is not suitable for further analysis i.e. to write down a supersymmetric and κ symmetric invariant action of N Dp-branes in co-dimension d. So, we consider the simpler cases of co-dimension one and two only.

3 N D0 branes in codimension one

The bosonic part of the action that follows from eq. (10) for N D0 branes in codimension one, i.e. in space-time dimension two, is

$$S = -T_0 \int d\tau STr \sqrt{-[\dot{x}^\mu \dot{x}^\nu \eta_{\mu\nu} + 2\dot{x}^1 \dot{\phi}^1 + (\dot{\phi}^1)^2]}, \quad (11)$$

where T_0 is the tension of a D0 brane, and τ is the world line parameter and $\cdot$ denotes its derivative. It is very easy to see that the action eq. (11) is world line reparametrisation invariant, hence, the corresponding first class constraint would be some generalization of the mass shell condition for $\phi = 0$.

Let p_μ be the momentum associated to x^μ. Before, we start to write down the canonical momenta associated to x^μ and ϕ, we rewrite the action eq.(11) in a suggestive manner.

$$S = -T_0 \int d\tau STr \sqrt{(\dot{x}^0)^2 - (\dot{\Phi})^2}, \quad (12)$$

where $\Phi = x^1 I + \phi^1$, from eq. (7), and the momenta associated to x^0 and Φ are

$$p_0 = -T_0 \dot{x}^0 STr \left[\sqrt{(\dot{x}^0)^2 - (\dot{\Phi})^2}\right]^{-1}, \quad (13)$$

and

$$p_\Phi = T_0 \dot{\Phi} \left[\sqrt{(\dot{x}^0)^2 - (\dot{\Phi})^2}\right]^{-1}. \quad (14)$$

The momentum $p_\Phi = \frac{1}{N} p_1 I + p_\phi$, with $p_\Phi \in U(N)$ and $p_\phi \in SU(N)$, where

$$p_1 = T_0 STr \ [\dot{\Phi}(\sqrt{(\dot{x}^0)^2 - (\dot{\Phi})^2})^{-1}]. \quad (15)$$

The mass shell condition for N D0 brane in spacetime dimension two is

$$-(p_0)^2 + \left(STr[\sqrt{p_\Phi^2 + T_0^2 I}]\right)^2 = 0, \tag{16}$$

and this reduces to the usual mass shell condition for N D0 branes when $\phi = 0$. Hence, the first order bosonic part of the action is

$$S = STr \int d\tau \Big[\frac{1}{N} p_\mu \dot{x}^\mu + p_\phi \dot{\phi} - \frac{e(\tau)}{2N}[-(p_0)^2 + \left(STr[\sqrt{p_\Phi^2 + T_0^2 I}]\right)^2]\Big], \tag{17}$$

where $e(\tau)$ is the Lagrange multiplier for the mass shell condition as given in eq. (16).

We can now supersymmetrise the above bosonic action by introducing a two component Majorana spinor field

$$\theta^\alpha = \begin{pmatrix} \theta^1 \\ \theta^2 \end{pmatrix}, \tag{18}$$

for $\phi = 0$. In the first order formalism the supersymmetric invariant action is

$$S = \int d\tau \quad [p_\mu(\dot{x}^\mu + i\bar{\theta}\gamma^\mu\dot{\theta}) - \frac{e(\tau)}{2}(p_\mu p^\mu + m^2) + m\bar{\theta}\gamma^2\dot{\theta}], \tag{19}$$

where $\bar{\theta} = \theta^T \gamma^0$ and the form of the γ matrices are

$$\gamma^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad \gamma^2 = \gamma^0\gamma^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{20}$$

The corresponding Clifford algebra is

$$\{\gamma^a, \gamma^b\} = -2\eta^{ab} \quad \text{with} \quad \eta^{ab} = (-,+). \tag{21}$$

The action eq. (19) is invariant under both global supersymmetry and local κ-symmetry transformations, and the form of transformations are

$$\delta_\epsilon \theta = \epsilon, \quad \delta_\epsilon x^\mu = -i\bar{\epsilon}\gamma^\mu\theta, \quad \delta_\epsilon(others) = 0, \tag{22}$$

$$\delta_\kappa x^\mu = i\delta_\kappa\bar{\theta}\gamma^\mu\theta, \quad \delta_\kappa e = 4i\bar{\kappa}\dot{\theta}, \quad \delta_\kappa\theta = (\gamma^\mu p_\mu - im\gamma^2)\kappa, \quad \delta_\kappa(others) = 0. \tag{23}$$

The corresponding supersymmetric and kappa symmetric invariant action for $\phi \neq 0$ is

$$S = STr \int d\tau \left(\frac{1}{N} p_\mu(\dot{x}^\mu + i\bar{\theta}\gamma^\mu\theta) + p_\phi\dot{\phi} - \frac{e}{2}[p_\mu p^\mu + \frac{M^2(p_\phi, p_1)}{N^2}] + \frac{M(p_\phi, p_1)}{N}\bar{\theta}\gamma^2\dot{\theta} - i\bar{\psi}\dot{\psi}\right), \tag{24}$$

where $\psi \in SU(N)$ and $M^2(p_\phi, p_1) = (STr\sqrt{p_\Phi^2 + T_0^2 I})^2 - p_1^2$. Where the supersymmetry transformations are same as eq. (22) and the kappa symmetry transformations are

$$\delta_\kappa x^\mu = i\delta_\kappa\bar{\theta}\gamma^\mu\theta, \quad \delta_\kappa e = 4i\bar{\kappa}\dot{\theta}, \quad \delta_\kappa\theta = (\gamma^\mu p_\mu - i\frac{M(p_\phi, p_1)}{N}\gamma^2)\kappa, \quad \delta_\kappa(others) = 0. \tag{25}$$

In the next section we extend the above construction for the case of codimension two and demonstrate how the same become more complicated.

4 N DO branes in codimension two

The bosonic part of the action that governs the dynamics of N D0 branes in codimension two is given by

$$S = -T_0 STr \int d\tau \left[\sqrt{(\dot{x}^0)^2 (detQ_{ij}) - (detQ_{ij}) \dot{\Phi}^k \dot{\Phi}^l Q^{-1}_{kl}}\right], \tag{26}$$

where i, j of determinant Q_{ij} can take only two values, namely, 1, 2. Evaluating the determinant in three space-time dimensions, give rise to

$$S = -T_0 STr \int d\tau \sqrt{\left(\dot{x^0}^2 - \dot{x^1}^2 - \dot{x^2}^2 - \dot{\phi^1}^2 - \dot{\phi^2}^2 - 2\dot{x^1}\dot{\phi^1} - 2\dot{x}^2\dot{\phi}^2 - \dot{x^0}^2 [\phi^1, \phi^2]^2\right)}, \tag{27}$$

using eq. (7), we can rewrite the above equation as

$$S = -T_0 STr \int d\tau \sqrt{(\dot{x^0}^2 - \dot{\Phi^1}^2 - \dot{\Phi^2}^2 - \dot{x^0}^2 [\Phi^1, \Phi^2]^2)}. \tag{28}$$

The momenta conjugate to x^0, Φ^1, Φ^2 are

$$\begin{aligned} p_0 &= -T_0 \dot{x}^0 STr \left(\frac{1 - [\Phi^1, \Phi^2]^2}{\sqrt{\dot{x^0}^2 - \dot{\Phi^1}^2 - \dot{\Phi^2}^2 - \dot{x^0}^2 [\Phi^1, \Phi^2]^2}} \right), \\ p_{\Phi^1} &= \frac{T_0}{2} \left\{ \dot{\Phi}^1, \frac{1}{\sqrt{\dot{x^0}^2 - \dot{\Phi^1}^2 - \dot{\Phi^2}^2 - \dot{x^0}^2 [\Phi^1, \Phi^2]^2}} \right\}, \\ p_{\Phi^2} &= \frac{T_0}{2} \left\{ \dot{\Phi}^2, \frac{1}{\sqrt{\dot{x^0}^2 - \dot{\Phi^1}^2 - \dot{\Phi^2}^2 - \dot{x^0}^2 [\Phi^1, \Phi^2]^2}} \right\}. \end{aligned} \tag{29}$$

The momenta $p_{\Phi^i} = \frac{1}{N} p_i I + p_{\phi^i}$, with $p_{\Phi^i} \in$ U(N) and $p_{\phi^i} \in$ SU(N). The mass shell condition for the N D0 brane in codimension two is (see the appendix for details)

$$-p_0^2 + \left[STr \sqrt{(p^2_{\Phi^1} + p^2_{\Phi^2} + T_0^2)(1 - [\Phi^1, \Phi^2]^2)} \right]^2 = 0. \tag{30}$$

It is easy to see that this mass shell condition reduces to the mass shell condition for codimension one, eq. (16), for either $\Phi^1 = 0$, or $\Phi^2 = 0$.

Let's rewrite eq. (27) in the first order formalism as

$$S = STr \int d\tau \Big[\frac{1}{N} p_\mu \dot{x}^\mu + p_{\phi^1} \dot{\phi}^1 + p_{\phi^2} \dot{\phi}^2 - \frac{e(\tau)}{2N} (-p_0^2 + \Big(STr \sqrt{(P^2_{\Phi^1} + P^2_{\Phi^2} + T_0^2)(1 - [\Phi^1, \Phi^2]^2)} \Big)^2) \Big] \tag{31}$$

The corresponding supersymmetric and kappa symmetric invariant action is

$$\begin{aligned} S &= STr \int d\tau \;\; \{ \frac{1}{N} p_\mu (\dot{x}^\mu + i\bar{\theta}\gamma^\mu\theta) + p_{\phi^1}\dot{\phi}^1 + p_{\phi^2}\dot{\phi}^2 - \frac{e}{2}[p_\mu p^\mu + \frac{\mathcal{M}^2(p_{\phi^1}, p_{\phi^2}, p_1)}{N^2}] \\ &\quad + \frac{\mathcal{M}(p_\phi, p_{\phi^2}, p_1)}{N} \bar{\theta}\gamma^2\dot{\theta} - i\bar{\psi}^1\dot{\psi}^1 - i\bar{\psi}^2\dot{\psi}^2 \}, \end{aligned} \tag{32}$$

where $\psi^1, \psi^2 \quad \in \quad SU(N)$ and

$$\mathcal{M}^2(p_{\phi^1}, p_{\phi^2}, p_1) = \left(STr\sqrt{(p_{\Phi^1}^2 + p_{\Phi^2}^2 + T_0^2 I)(1 - [\Phi^1, \Phi^2]^2)}\right)^2 - p_1^2 - p_2^2.$$

The corresponding supersymmetric and kappa symmetric transformations are same as eq. (22), and eq. (25) except that M has to be replaced by $\mathcal{M}$ in the kappa symmetry transformations, and the definition of $\bar{\theta}$ and γ matrices are given in section 3.

5 Conclusion

In this note, we have examined Sorokin's prescription to see the coincident N D0 branes as a single ND0-brane in different co-dimensions to get the supersymmetric and kappa symmetric invariant action in the first order formalism. In particular, we have considered only the N D0 branes in co-dimension one and two in flat spacetime. To implement the same prescription for higher branes and for higher codimensions one needs to know the corresponding mass shell conditions which become more complicated and the analysis becomes more involved (as seen in the appendix) though in principle it is possible. Moreover, even in this prescription we still encounter the usual problem of the nature of 'trace' one should adopt. However, the prescription by itself is beautiful since the kappa symmetry is reduced to an abelian symmetry and once we get the correct mass shell condition for a certain system, it is straight forward to write down the corresponding supersymmetric and kappa symmetric invariant action.

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6 Appendix A

In this section we shall derive the mass shell condition eq. (30) by expanding it term by term. For this purpose we define some notations.

$$\begin{aligned} &1 - [\Phi^1, \Phi^2]^2 \equiv Z \equiv 1 - B, \quad \Rightarrow B = [\Phi^1, \Phi^2]^2, \\ &\dot{\Phi^1}^2 + \dot{\Phi^2}^2 \equiv Y, \\ &p_{\Phi^1}^2 + p_{\Phi^2}^2 \equiv A. \end{aligned} \tag{33}$$

The momentum p_0 in these notations becomes

$$p_0 = -T_0 \dot{x}^0 STr\left(\frac{Z}{\sqrt{\dot{x^0}^2 Z - Y}}\right), \tag{34}$$

and expanding it, we get

$$p_0 = -T_0 STr\ [Z^{\frac{1}{2}} + \frac{1}{2}\frac{Z^{\frac{-1}{2}}Y}{\dot{x^0}^2} + \frac{3}{8}\frac{Z^{\frac{-3}{2}}Y^2}{\dot{x^0}^4} + \ldots].^3 \tag{35}$$

Now the other term present in the mass shell condition becomes, in these notations

$$STr\sqrt{(p^2_{\Phi^1} + p^2_{\Phi^2} + T_0^2)Z} = STr\sqrt{(A + T_0^2)Z}, \tag{36}$$

which after expanding takes the form

$$T_0 STr Z^{\frac{1}{2}} + \frac{1}{2T_0}STr(Z^{\frac{1}{2}}A) - \frac{1}{8T_0^3}STr(Z^{\frac{1}{2}}A^2) + \ldots. \tag{37}$$

The second term $\mathrm{STr}(Z^{\frac{1}{2}}A)$ and the 3rd term $\mathrm{STr}(Z^{\frac{1}{2}}A^2)$ of the above equation can be written as

$$\begin{aligned}
\frac{1}{2T_0}STr[Z^{\frac{1}{2}}A] &= \frac{T_0}{2\dot{x^0}^2}STr(Z^{\frac{-1}{2}}Y) + \frac{T_0}{2\dot{x^0}^4}STr(Z^{\frac{-3}{2}}Y^2) + \ldots, \\
-\frac{1}{8T_0^3}STr(Z^{\frac{1}{2}}A^2) &= -\frac{T_0}{8\dot{x^0}^4}STr(Z^{\frac{-3}{2}}Y^2) + \ldots,
\end{aligned} \tag{38}$$

substituting these expressions of the second term and the third term in eq.(37), we see that it becomes exactly the -ve of the eq.(35). Which upon substituting in eq. (30) gives us the right hand side. Hence, the mass shell condition is proved.

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